# Two Step Continuous Trigonometrically Fitted Method for Solving Oscillatory second Order Ordinary Differential Equations

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**Abstract:** The Continuous Two Step Trigonometrically-Fitted Second Order Method (TSTSOM) is used in this study to solve an oscillating problem of ordinary differential equations. The coefficients of the developed approaches are determined by the approximate solution's frequency and step size, a discrete trigonometrically -fitted second order ordinary differential equation was recovered as a by-product. To demonstrate the method's usefulness and efficiency, the method's stability and other properties qualities will be described and implemented to solve linear and nonlinear initial value oscillatory problems.

**Keywords:** Linearmultistep, interpolationtechniques, Trigonometric-fitted, predictor-corrector.

## 1. Introduction

Mathematical modelling isacrucialtechnique for analyzing wide rangeofreal-world problems involving differential equations, spanning from physics, meteorology, and engineering to chemistry, biology, and social sciences. Differential equations are equations in which the dependent and independent variables have differential coefficients. Ordinary and partial differential equations are the two types of differential equations. Ordinary differential equations (odes) are differen-tiale equations in which the unknown parameter is afunction of one independent variable, whereas partial differential equations are those involving two or more independent variables (pdes).In Science and Engineering usually, mathematical models are developed to help in the understanding of physical phenomena. These models often yield equations that contains some derivative of an unknown function of one or several variables. In what follow, we consider a numerical solution of general second-order IVP soft he form

$$y'' = f(x, y, y'), y(x_0) = y_0, y'(x_0) = y'_0, x \in [x_0, x_n] \quad (1)$$

where *f* satisfies the Lipschitz theorem.

For the solution of a number of problems, numerical approaches based on the usage of polynomial functions have been presented (1).Adeniran and Ogundare(2015) offered a block hybrid technique for the direct integration of second order IVP whose solutions oscillate, and Ngwane and Jator (2013) proposed a hybrid block method for the system of first order IVP including oscillatory problems. Sanugi and Evans proposed the leap frog approach and the Runge-Kutta method, whereas Neta (1986) constructed families of backward differentiation equations. All of these techniques were implemented in a step-by-step manner.

Despite their success, these methods have some drawbacks, including sensitivity to frequency changes, the necessity that the Jacobean's Eigen values be wholly magi-nary, and computing burden.

Psihoyios and Simos (2003 and 2005) proposed trigonometrically fitted schemes for the solution of oscillatory problems that are applied in predictor-corrector mode based on the well-known Adams-Bash forth method as predictor and Adams-Moulton as corrector, in the spirit of Kayode and Adegboro (2018) proposed predictor-corrector for solving second order ordinary differential equations.The method sarevery expensive to implement, require more human labor,andhavea lower level of accuracy. The purpose of this study is to develop a Discrete Trigonometrically Fitted Second Method (DTSM).

This is accomplished by firstestablishinga TSCTM, which then gives adiscrete method that is used as a DTSMand uses the solution's frequency as a priori knowledge. TSCTM, in particular, is made up of a collection of continuous functions, whereas DTSM is a by-product of TSCTM. Because the coefficients of the Continuous Trigonometric Second method TSCTM are functions of frequency and step size, the suggested technique's solutions are extremely accurate if (1) has periodic solutions with known frequencies. The TSDM is utilized to produce the approximation $y_{n+1}$to the exact solution $(x_{n+1})$ on the interval $[x_n, x_{n+1}]$, as described in (Ngwane and Jator 2013; Ng-wane and Jator 2014). $h = x_{n+1}x_n$, $h = x_{n+1}x_n$, $h = x_{n+1}x_n$, On a partition $[a, b]$, $n = 0, . . . , N1$, where $a, b \epsilon R, h$ is the constant step size, $n$ is a grid indexand$N$>0isthenumberofsteps

Thefollowingisa breakdown of the paper's structure.Wee stablishatrigono metric basisre presentation$U(x)$ forth exact solution$y(x)$in Section1.1.1" Derivation of the Method". create a TSCTM for problem-solving (1). The TSCTM's error analysis and stability are detailed in Section 1.2" Error analysis and stability". Section 1.3 "Numerical examples" contains numerical examples that demonstrate the TSTSOM's accuracy and efficiency. Finally, Section 1.4" Conclusion, "we make some closing remarks.

**1.1.1 Derivation of the Method.**

Two Steps continuous Trigonometrically-fitted Methods (TSCTM) is obtained by approximating the exact solution $y(x)$ by searching the solution $y(x, u)$, which resultsinadiscretemethodasaby-product. The type of method is

$$y(x,u) = \sum_{j=0}^{k} a_j x^j + a_{k+1} sin(wx) + a_{k+2} cos(wx) \quad (2)$$

Will be used as abasis function to approximate the solution of the second order initial value problems of the form

Thesecond derivative of (2) is given as:

$$y''(x,u) = \sum_{j=0}^{k} j(j-1) a_j x^{j-2} - w^2 a_{k+1} sin(wx) - w^2 a_{k+2} cos(wx) \quad (3)$$

Through interpolation of (2)at $x_{n+j}, j$=0,1, collocation of (3) at $x_{n+j}$, $j$ =0(2)$k$ to obtain 2$k$+1system of equation

$$\begin{aligned} y(x_{n+j}, u) &= y_{n+j}, j = 0(1) \\ \frac{d^2}{dx^2} y(x_{n+j}, u) &= f_{n+j}, j = 0(1)k \end{aligned} \quad (4)$$

Solving the system equation (4) by Crammer's rule to obtain $a_j$, $j$ = 0, 1, 2, 3, 4.Our continuous TSCTM is constructed by substituting the values of $a_{j's}$ into equation (2).After some algebraic manipulation, the TSCTM is expressed in the form

$$y_{n+k} = a_n(x,w) + a_{n+1}(x,w) + h^2(\beta_n(x,w) f_n + \beta_{n+1}(x,w) f_{n+1} + \beta_{n+2}(x,w) f_{n+2} \quad (5)$$

$w$ is the frequency ,$a_n(w,x), a_{n+1}(w,x), \beta_n(w,x), \beta_{n+1}(w,x), \beta_{n+2}(w,x)$ are continuous coefficients. The continuous coefficients in equation (5) is used to generate the method of the form in Equation (2). Thus, evaluating (5) at $x = x_{n+2}$ and letting $u=wh$,we obtain the coefficients of (2) as follows

$$\beta 0 = \frac{(2\cos(u) + u^2 \quad \_ \quad 2)\sin(u)}{((-2(\cos(u)-1)\sin(u)))^2} \quad (6)$$

$$\beta 1 = \frac{\cos(u)u^2 + 2\cos(u)2 \quad \_}{2(\cos(u))^2 u - 2(\cos(u))^2 + 2\cos(u) - u - 1}$$

$$\beta_2 = \frac{-u^2 - 2\cos(u) + 2}{2\cos(u) - 2}$$

### 1.2 Error Analysis and Stability

Local Truncation Error:

The Taylorseries is used for small values of u (seeSimos(2007)).Thus the coefficients in equation (6) can be expressedas

$$\beta_0 = \frac{1}{12}u^2 + \frac{1}{240}u^4 + \frac{1}{6048}u^6 + \frac{1}{172800}u^8 + \frac{1}{5322240}u^{10} + \frac{691}{118879488000}u^{12}$$

$$\beta_1 = 4 - \frac{5}{6}u^2 + \frac{1}{120}u^4 + \frac{1}{3024}u^6 + \frac{1}{86400}u^8 + \frac{1}{2661120}u^{10} + \frac{691}{59439744000}u^{12}$$

$$\beta_2 = \frac{1}{12}u^2 + \frac{1}{240}u^4 + \frac{1}{6048}u^6 + \frac{1}{172800}u^8 + \frac{1}{5322240}u^{10} + \frac{691}{118879488000}u^{12}$$

(7)

For practical computations when u is small, it is advisable to use the series expansion (7).ThustheLocalTruncationErrorfor method (6) subject to equation (7)is obtained as

$$\frac{h^6}{6048}\left(w^2y^4x_n + y^6(x_n)\right) + 0^8$$

$$\frac{h^6}{3024}\left(w^2y^4x_n + y^6(x_n)\right) + 0^8$$

$$\frac{h^6}{6048}\left(w^2y^4x_n + y^6(x_n)\right) + 0^8$$

The local truncation error are $\left(\frac{1}{6048}, \frac{1}{3024}, \frac{1}{6048}\right)$ and is of at lease order

**Stability Properties**

**Proposition 1**. The trigonometrically-fitted second derivative method (7) is applied to a test equation $y^{rr}=-\lambda^2 y$,where $\lambda$ is are al constant (see Simos 2002), it yields

$$y'' = M(\gamma^2; u)y_{n+1}, y = h\lambda; u = kh \tag{8}$$

with

$$M(\gamma^2; u) = \frac{A_0 + \gamma^2\beta_0}{A_1 - \gamma^2\beta_1} \tag{9}$$

(10)

Where the matrix $M$ ($\gamma^2$; $u$) is the amplification matrix which determines the stabilityofthemethod.

**Proof**. We begin by applying (6) to the test equation $y^{rr}=\lambda^2 y$ respectively, by letting $\gamma = h\lambda$, $u = kh$, we obtain a linear equation which is used to solve for $y_{n+2}$ with(9)as consequence.

**Definition 1**. A region of stability is a region in the $\gamma$-$u$ plane, in which therationalfunction$M(\gamma^2; u) \leq 1$

**Definition2**.The method (5)is zero stable provided the root of the first characteristics polynomial have modulus less than or equal to unity and those of modulus unity are simple (Lamberts1973).

**Definition 3**. Method (5) is consistent if it has order $p$ >1 (Kayode and Adegboro (2018). The TSCTM is consistent as it has order $p$ >1 and zero stable,

Hence convergent. Since Convergence=Zero stability consistency

**Linear Stability and Region of Absolute Stability of the Method** Themethod is zero stable provided the roots $R_j, j$=1,2,3of the first characteristic polynomial $\varrho(R)$s pecified by

$\varrho$(R)=$r^2$−2$r$+1=0for$r$=1,1and the multiplicity does not exceed1(seeJatoretal.(2019)).

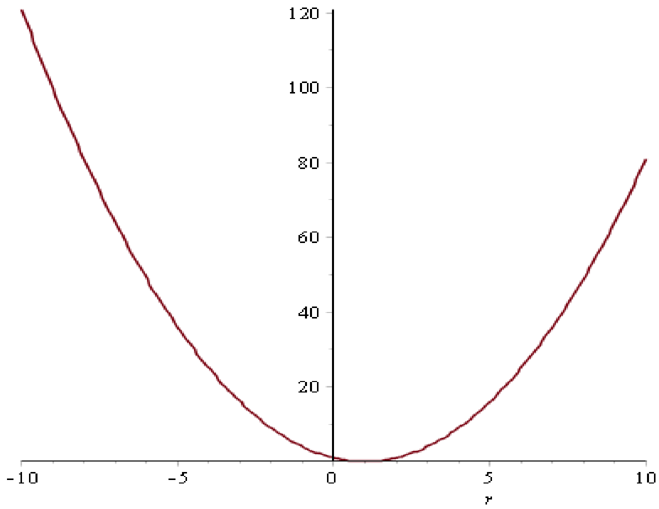


**Figure1:2 D plot for Zero Stability Result showing the accuracy of the new method TSCTM**

**Region of Absolute Stability of the Method**

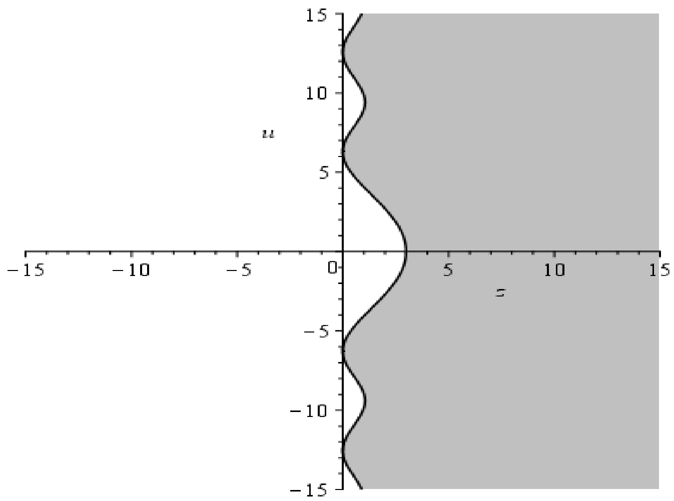


**Figure 2: Region of Absolute Stability for TSCTM**

**1.3** Numerical Examples

The performance and accuracy of the newly developed TSCTM are discussed in this part for a range of well-known oscillatory IVPs, both linear and nonlinear issues. For the computation, the fitting frequency of each problem is utilized as the default frequency. The approximation solutions' absolute errors or maximum errors are estimated and compared to results from existing approaches in the litera-ture. $r(t)$ represents an error of the kind $rast10t$. All calculations were carried out using written Maple 2016 and 2017 codes, which were run on a Windows 8.1computer.

Example1

$$y'' = -100 + 99\sin(x), y(0) = 1, y'(0) = 11, x \in [0,1000],$$

$$h = \frac{1}{3200}, w = 5000$$

wheretheanalyticalsolutionisgivenby

$y(x) = cos10x + sin(10x) + sinx$

**Table1: Table forthenewmethod**

| N | y-exact | y-computed | ErrorsinTSCTM |
|---|---|---|---|
| 1000 | 1.00206074108828 | 1.00206191363586 | 1.17254758e−6 |
| 2000 | 1.00171752866070 | 1.00171840801032 | 8.7934962e−7 |
| 4000 | 1.00137480457620 | 1.00137480457623 | 5.8615198e−7 |
| 8000 | 1.00103081040944 | 1.00103120115695 | 3.9074751e−7 |
| 16000 | 1.00068730464678 | 1.00068749999022 | 1.9534344e−7 |
| 32000 | 1.00034370116678 | 1.00034379883829 | 9.767151e−8 |

Table 1 shows the results of problem 1 when computed with the method. The iteration (N) was increased from 1000 to 32000. The results show that increase initerationimprovestheaccuracyofthemethod.

Example2

ConsidertheScalartestequation

$$y'' = w^2 y, y(0) = 1, y'(0) = 0, w = 10, h = \frac{\pi}{200}$$

Exact$y(x) = coswx$

**Table 2: Result showing the accuracy of the new method TSCTM**

| x | y-exact | y-computed | ErrorsinTSCTM |
|---|---|---|---|
| $5\pi$ | 0.999876521928723 | 0.999999999999992 | $2.469713e{-}05$ |
| $10\pi$ | 0.999506118208559 | 0.999999999999992 | $4.938817e{-}05$ |
| $15\pi$ | 0.998888880312983 | 0.999876491416312 | $9.876111e{-}05$ |
| $20\pi$ | 0.998024960672684 | 0.999506057182589 | $1.481096e{-}04$ |
| $25\pi$ | 0.996914572637924 | 0.999135577193974 | $2.221005e{-}04$ |
| $30\pi$ | 0.995557990425848 | 0.998518354528637 | $2960364e{-}04$ |

Table2 shows the computed result for the new method, error signifies the efficiency of the new method solved with problem 2.

Example3

$$y'' = w^2 y, y(0) = 1, y'(0) = 2, w = 10, h = \frac{\pi}{200}$$

Exact $y(x) = sinwx$

In comparison to $3N + 1$ and 4N function evaluations in N steps, this requires only$3N + 1$ function evaluations in N stages. For example, in the continuous scheme, if$n = 0$, $y1$ is obtained on the subinterval $[x_0; x_1]$, because $y0$ is known from the IVP; similarly, if $n = 1$, $y_2$ is obtained on the sub interval $[x_1; x_2]$, because $y_1$ is

Known from the previous computation, and soon until were ach the final sub interval. As a result, this strategy is more effective.

**Table3: Result showing the accuracy of TSCTM**

| x | y-exact | y-computed | Errorsin TSCTM |
|---|---|---|---|
| $5\pi$ | 0.996733651293647 | 0.996980234565711 | $2.465833e{-}05$ |
| $10\pi$ | 0.993221153089616 | 0.993714258729819 | $4.931056e{-}05$ |
| $15\pi$ | 0.989463372820915 | 0.990447879558909 | $9.845067e{-}04$ |
| $20\pi$ | 0.985461238494462 | 0.986936903191648 | $1.475665e{-}03$ |
| $25\pi$ | 0.981215738461914 | 0.983425493233487 | $2.209755e{-}03$ |
| $30\pi$ | 0.976727921175581 | 0.979671220300475 | $2.943299e{-}03$ |

Table 3 shows the computed result for the new method, error signifies the effi-ciency of the new method solved with problem 3.

Example 4

$$y'' = \begin{pmatrix} 2498 & 4998 \\ -2499 & -4999 \end{pmatrix} y(t), y(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, y'(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, 0 \leq t \leq 100$$

Exact solution :$(2cos(t), -sin(t))^t$

**Table4: Result showing the accuracy of TSCTM**

| N | y-exact | y-computed | ErrorsinTSCTM max$\|\lfloor y_i - y(x_i)\|\|$ |
|---|---|---|---|
| 10 | 1.99999900000008 | 2.000000000000630 | $6.3e{-}13$ |
| 40 | 1.99999600000133 | 2.000000000000274 | $2.7e{-}12$ |
| 80 | 1.99999900000675 | 1.999999000000100 | $1.0e{-}12$ |
| 120 | 1.99998400002133 | 1.999996006024520 | $4.5e{-}12$ |
| 180 | 1.99997500005208 | 1.999995009029670 | $2.9e{-}12$ |
| 220 | 1.99996400010800 | 1.999984012108890 | $8.8e{-}11$ |

Table 4 above shows the computed result for the new method TSCTM, error signifies the efficiency of the new method solved with problem 4

Example5

(PeriodicProblem)VandeVyver

$$y_1'' = y_1 + \frac{1}{100}\cos(x), y_1(0) = 1, y_1'(0) = 1$$
$$y_2'' = y_2 + \frac{1}{100}\sin(x), y_2(0) = 0, y_1'(0) = 0.9995, x_{end} = 10$$

With the theoretical solution :$y_1(x) = cos(x) + 0.0005xsin(x), y_2(x) = sin(x)0.0005cos(x)$ –

**Table5: Result showing the accuracy of TSCTM**

| N | y-exact | y-computed | TSCTM max$\|\lfloor y_i - y(x_i)\|\|$ |
|---|---|---|---|
| 0.1 | 0.999999500500042 | 0.000999498751775049 | $4.98251761e{-}07$ |
| 0.2 | 0.999998002000666 | 0.00199899950324900 | $1.99750245e{-}06$ |
| 0.3 | 0.999995504503368 | 0.299849910609906 | $4.49460223e{-}06$ |
| 0.4 | 0.999992008010646 | 0.399800270764537 | $7.99469593e{-}06$ |
| 0.5 | 0.999987512525990 | 0.00499750516068165 | $1.24926327e{-}05$ |
| 0.6 | 0.999982018053892 | 0.599701361141322 | $1.79955544e{-}05$ |
| 0.7 | 0.999975524599842 | 0.00699652091374632 | $2.44963093e{-}05$ |
| 0.8 | 0.999968032170325 | 0.00799603621277948 | $3.20040361e{-}05$ |
| 0.9 | 0.999959540772827 | 0.00899555036352342 | $4.05095821e{-}05$ |
| 1.0 | 0.999950050415832 | 0.00999507450998187 | $5.00240831e{-}05$ |

Table 5 shows the computed result for the new method TSCTM, error signifies the efficiency of the new method solved with problem 5.

**Table 6: Comparison of the new error with Simon.Ngwaneand Jator (2013)**

| N | Simos (1998) | NgwaneandJator(2013) | newmethodTSCTM |
|---|---|---|---|
| 1000 | 1.4$e$−1 | 2.14$e$−03 | 8.83$e$−04 |
| 2000 | 3.4$e$−2 | 5.98$e$−05 | 1,17$e$−05 |
| 4000 | 1.1$e$−3 | 2.06$e$−05 | 1.17$e$−06 |
| 8000 | 8.4$e$−5 | 1.26$e$−06 | 8.79$e$−07 |

Table 6 shows the comparison for computed error for the new method TSCTM, error signifies the efficiency of the new method solved with problem 1.

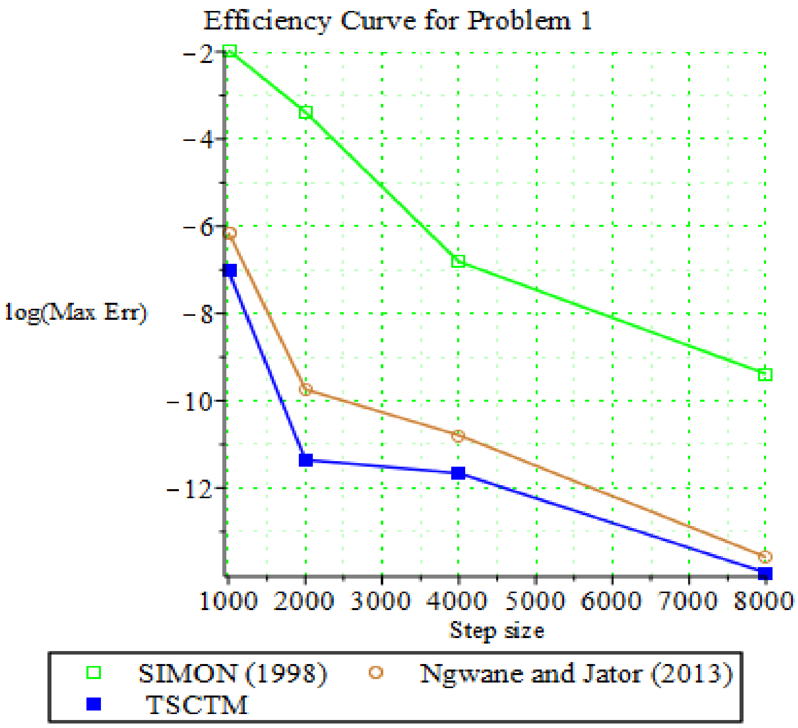


Figure3: Efficiency curve k=2forexample1

**Table 7:Comparison of the new error with AliShorki(2012).**

| x | AliShorki (2012) | TSCTM |
|---|---|---|
| $5\pi$ | 2.3659$e$−04 | 2.469713$e$−05 |
| $10\pi$ | 5.1547$e$−04 | 4.938817$e$−05 |
| $15\pi$ | 6.2689$e$−04 | 9.876111$e$−05 |
| $20\pi$ | 8.3654$e$−04 | 1.481096$e$−04 |

Table 7 above shows the comparison table error for the new method TSCTM,error signifies the efficiency of the new method solved with problem 3 over existing method.

**Table8: Comparison of the new error for example 4 with Nguyenetal.(2007).**

| x | Nguyen*etal.*(2007) | TSCTM |
|---|---|---|
| 10 | 3.3*e*−12 | 6.3*e*−13 |
| 40 | - | 2.7*e*−12 |
| 43 | 0.9−11 | - |
| 80 | 3.7*e*−12 | 1.0*e*−12 |

Table 8 above shows the comparison table; error for the new method, error signifies the efficiency of the new method solved with problem 4 over existing method.

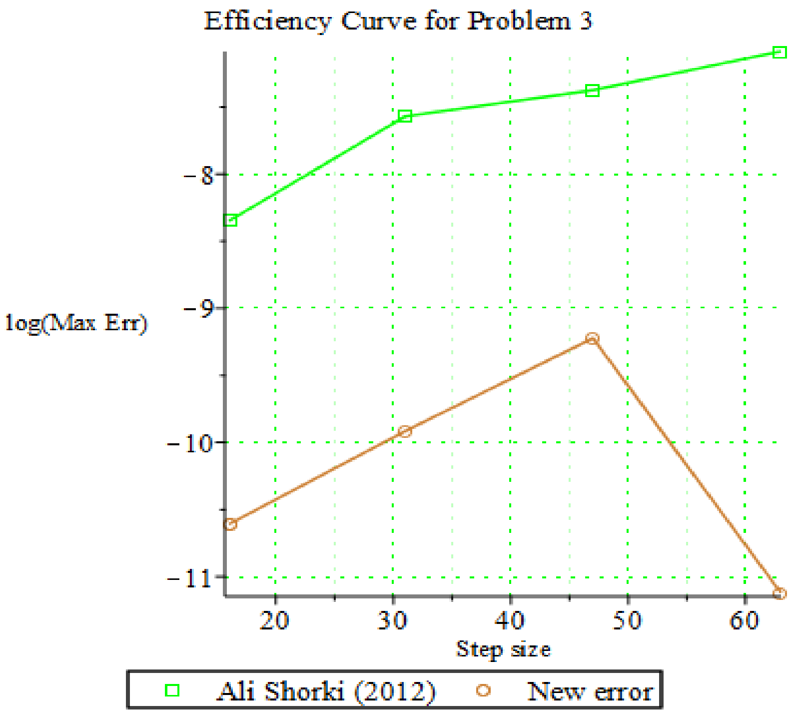


**Figure 4: Efficiency curve for example 3**

**Table9: Comparison of the new error for example 5 with Nguyen*etal*.(2012)**

| x | Nguyen. *etal*(2007) | x | TSCTM |
|---|---|---|---|
| 73 | | 10 | 6.3*e*−13 |
| 143 | 9.0−12 | 43 | 2.7*e*−12 |
| 170 | 3.7*e*−12 | 80 | 1.0*e*−12 |

Table 9 above shows the comparison table, error for the new method signifies the efficiency of the new method solved with problem 5 over step length of the existing method.
Example 6

**ResonanceVibration of a Machine**

A stamping machine applies hammering forces on metal sheets by a die attached to the plunger which moves vertically up and down by a fly wheel makes the impact force on the metal sheet and there fore the supporting base, intermittent and cyclic. The bearing base on which the metal sheet is situated has a mass, $M = 2000kg$.The force acting on the base follows a function'$s(t) = 2000sin(10t)$,in whicht=time in seconds. The base is supported by an elastic pad with an equivalent spring constant $k = 2 * 10^5 N/M$. Determine the differential equation for the instant a neous position of the base $y(t)$ if the baseisinitially depressed down by an Amount 0.1m.

**Solution:**The mass-spring system above is modeled as differential equation: The Bearing base mass=2000kg

Spring constant $k = 2*10^5 N/m$

i.e.$ma = my'' = 2000sin(10t)$;where$a = y''$

Force (ma) on the metal sheet=$m\frac{d^2y}{dt^2} = my''$

Initialconditions on the system are

$y(t_0) = y_0; \frac{dy}{dt} | t=0 = y'(t_0) = y'(0); t_0 = 0, y'(t_0) = 0.1$

There fore, the governing equation for the instant aneous position of the basey(t)isgivenby

$My'' + ky = F(t); y(t_0) = y_0, y'(t_0) = y_0$

**The oretical solute on:**

$$\frac{1}{10}\cos(10)t + \frac{1}{200}\sin(10)t - \frac{t}{10}\cos(10)\,t,,$$

**Table10: Table for problem 7 TSCTM, showing the accuracy of the new method**

| N | y-exact | y-computed | ErrorsinTSCTM $\max\|\lfloor y_i - y(x_i)\rfloor\|$ |
|---|---|---|---|
| 0.01 | 0.0999999500016710 | 0.100000099994178 | $1.499925070e{-07}$ |
| 0.02 | 0.0999998000134000 | 0.999999999996060 | $1.999862060e{-07}$ |
| 0.03 | 0.0999995500453379 | 0.0999999000292754 | $3.499839375e{-07}$ |
| 0.04 | 0.0999992001077328 | 0.0999996000212815 | $3.999135487e{-07}$ |
| 0.05 | 0.0999982003653984 | 0.0999993000387002 | $5.498277629e{-07}$ |
| 0.06 | 0.0999982003653984 | 0.0999987999664171 | $5.996010187e{-07}$ |
| 0.07 | 0.0999975505816685 | 0.0999982999203376 | $7.493386691e{-07}$ |
| 0.08 | 0.0999968008703942 | 0.0999975997301152 | $7.988597210e{-07}$ |
| 0.09 | 0.0999959512423277 | 0.0999968995646254 | $9.483222977e{-07}$ |
| 0.10 | 0.0999950017083162 | 0.0999959992026629 | $9.974943467e{-07}$ |

Table 10 shows the computed result for the new method TSCTM, error signifies the efficiency of the new method solved with problem 6.

## 1.4 CONCLUSION

We have presented a TSCTM for solving periodic IVPs with a non-self-stating algebraic 3 order. The method convergence and accuracy were established, and the approach was evaluated with several standard oscillatory problems and found to be accurate and favorably compare to other ways in literature, as shown in Tables1-10 above, with the exception of Example 2 and 6, which has no competition.

Competing interests

The authors declare that they have no competing interests.

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