Vector Autoregressive Model to Forecast Malaysian Economic Growth

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Abstract – This study presents a comparative study on univariate time series via the Autoregressive Integrated Moving Average (ARIMA) model and multivariate time series via Vector Autoregressive (VAR) model in forecasting economic growth in Malaysia. This study used monthly economic indicators price from January 1998 to January 2016. The aim is to evaluate a VAR and ARIMA model to forecast economic growth and to suggest the best time series model from an existing model for forecasting economic growth in Malaysia. The forecast performances of these models were evaluated based on an out-of-sample forecast procedure using an error measure, Mean Absolute Percentage Error (MAPE). The results revealed that the VAR model outperform ARIMA model in term of forecasting accuracy.

Keywords: univariate, multivariate, ARIMA, VAR, growth, accuracy, forecast

1.0 Introduction

Economy is a system of trade and industry by which the wealth of a country is made and used. Economic growth is an expansion in the limit of an economy to create goods and services, compared from one period of time to another. As indicated by [1], economy development assumes an imperative part of any nation, including Malaysia as it prompts increment in the way of life, pay per capital, business opportunities, job level, monetary security and other different things. Economic indicators measure how vigorous an economy of a country is. They can quantify particular divisions of an economy, for example, the lodging or retail division, or they give quantification or estimations of an economy in general, such as Gross domestic product or unemployment.

A time-series is a sequence of values measured over time, in discrete or continuous time units. Time series analysis is one of the main tools to predict the value of an economic variable with the suitable model to depict the time variety of historical data [2]. Time series models can be divided into two which are univariate models where the observations are those of single variable recorded successively over equivalent separated time intervals and multivariate models, where the observations are of multiple variables. A typical assumption in many time series techniques is that the data are stationary. Univariate time-series (UTS) refers to a time-series that consists of single observations recorded sequentially through time and are useful for analyzing the dynamic properties of time series and forecasting. In addition, the analysis is a way to introduce the tools necessary for analyzing more complicated models. Meanwhile, multivariate time series analysis (MTS) is an important statistical tool to study the behavior of time dependent data and figure future qualities relying upon the historical backdrop of varieties on the information. [3] Stated that multivariate time series analysis is employed once one needs to model and justify the interactions and co-movements among a group of time series variables. Multivariate methods are very important in economics and much less so in other applications of forecasting. The multivariate perspective is focal in financial matters, where single variables are generally seen with regards to relationship to different variables.

The univariate time series method is a way to deal with forecasts of a period arrangement on the basis of the historical behavior of the series itself. This technique is exceptionally helpful in light of the fact that it can give plausible precise short- to medium-term forecasts furthermore economical to apply [4]. The motivation for multivariate forecasting is that there is information in multiple economic time series that can be used to improve forecasts of the variable or variables of interest [5].

This study, therefore, aims to develop a model using Univariate and Multivariate Time Series to forecast economic growth in Malaysia based on time series data collected from World Bank Development Indicators and Ministry of Finance Malaysia.
2.0 Materials and Methods

This study will employ mainly secondary economic data in its analysis. All data to be used in the analysis is taken from World Bank Development Indicators and Ministry of Finance Malaysia from January 1998 to January 2016. Economic indicators used as the variable in this study are Currency in Circulation (CIC), Exchange Rate (EXC), External Trade (EXT) and Reserve Money (RM), see [3] and [5]. The data organized would be analyzed both descriptively and quantitatively. Charts such as trend graphs and tables were employed to aid in the descriptive analysis. UTS refers to a time-series that consists of single observations recorded sequentially through time and MTS analysis is used to model and explain the interactions and co-movements among a group of the series variables. Therefore, this study focus on the UTS technique via ARIMA model while MTS techniques via Vector Autoregressive (VAR). All estimations were carried out using Eview software.

Correlation Test: In this study, the correlation test is utilized to legitimize whether the variables can be utilized to forecast economic growth. To figure out if the relationship between variables is noteworthy, compare the p-value to the significance level. Conventionally, a significance level, denoted as α or alpha of 0.05 functions well. The p-value tells whether the relationship coefficient is significantly different from 0.

Unit Root Test: Techniques such as unit root test for each of the series is carried out to check the stationary position of the data. The unit root test is conducted by employing the Augmented Dickey-Fuller (ADF) test procedure for testing integrated order of (1) versus integrated order of (0). The ADF test:

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{j=1}^{p} \delta_j Y_{t-j} + \epsilon_t,$$  \hspace{1cm} (1)

where, $t$ is the time trend, $\alpha$ is an intercept constant, $\beta$ is the coefficient on a time trend, $\gamma$ is the coefficient presenting process root, $p$ is the lag order of the first-differences autoregressive process and $\epsilon_t$ is the white noise residual of zero mean and constant variance.

The unit root hypothesis of the ADF test can be rejected if the t-test statistic from these tests is negatively less than the critical value. In other words, for the ADF test, a unit root exists in the series if the null hypothesis of equal to zero is not rejected.

2.1 Autoregressive Integrated Moving Average (ARIMA) modeling

ARIMA model was initially proposed by [6]. It forecasts future estimations of a time series as a linear combination of its own past values and a series of errors, additionally called as random shocks. ARIMA models are constantly applied in univariate situations where time series show confirmation of non-stationarity by utilizing an initial differencing step to evacuate the non-stationarity as stated in [7].

In ARIMA model, the future value of a variable is a linear combination of past values and past errors, expressed as follows:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - ... - \theta_q \epsilon_{t-q},$$  \hspace{1cm} (2)

where $Y_t$ is the actual value and $\epsilon_t$ is the random error at $t$, $\phi_i$ and $\theta_i$ are the unknown coefficient, $p$ and $q$ are integers that are often referred to as autoregressive (AR) and moving average (MA), respectively, as stated by [8].

Model identification involves deciding the orders of the AR and MA parts of the model. Potential model will be recognized and described. It will identify value whether the variable, which is being forecast, is stationary in time series or not. Number of differencing ($d$) and autoregressive ($p$) and moving average ($q$) terms are determined by using Autocorrelation function (ACF) and partial autocorrelation function (PACF). According to [9], caution to be taken in differencing as overdifferencing will incline to increment in the standard deviation. The best suitable model for forecasting, relatively small of AIC (Akaike Information Criterion) developed by Hirotugu Akaike, [10] will be used in this study to determine the best ARIMA model. The AIC rule gives the best number of lags and parameters to be estimated in the models. Diagnostics is performed to check whether the fitted model is appropriate or not and to examine the validity of the fitted model. After all parameter has been estimated, it can be used to obtain forecasts value. The forecast model selected can be expressed as follows:
Impulse response standard errors are

\[ Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + \varepsilon_t \]  

(3)

where, \( \varepsilon_t = y_t - \hat{y}_t \) is the difference between the actual value of the series and the forecast value.

### 2.2 Vector Autoregressive (VAR) Modeling

The VAR model, proposed by [11], is one of the popular, adaptable and simple to use models for analysis of multivariate time series. It is applied to handle the mutual influence among multiple time series. VAR models elongate the univariate autoregressive (AR) model to dynamic multivariate time series by considering for more than one evolving variable. According to [7], all variables in a VAR model are dealt with symmetrically in a basic sense; each variable has an equation explaining its evolution based on its own lags and the lags of the other model variables.

Let \( y_t = (y_{1t}, y_{2t}, ..., y_{nt})' \) denote a \( n \times 1 \) vector of time series variables. A VAR model with \( p \) lags can then be expressed as follows:

\[ y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + ... + A_p y_{t-p} + \varepsilon_t \]  

(4)

Where, \( \varepsilon \) is a \( k \times 1 \) vector of constant, \( A_i \) is a time-invariant \( k \times k \) matrix and \( \varepsilon_t \) is a \( k \times 1 \) vector of error terms.

Johansen cointegration test is used to test for the order of integration is standard in applied econometric work. To answer this inquiry cointegration test created by [12] and [13] is employed as this procedure is known to be the most reliable test for cointegration. The Johansen - Juselius technique is as follows:

\[ \Delta X_t = \mu + \sum_{i=1}^r \Gamma_i \Delta X_{t-i} + \alpha BX_{t-i} + \varepsilon_t \]  

(5)

Where \( X_t = (n \times 1) \) vector of all the non stationary indices in our study, \( \Gamma = (n \times n) \) matrix of coefficient, \( a = (n \times r) \) matrix of error correction coefficient where \( r \) is the number of cointegrating relationships in the variable, so that \( 0 < r < n \). This measures the speed at which the variables adjust to their equilibrium, \( B = (n \times r) \) matrix of \( r \) cointegrating vectors, so that \( 0 < r < n \).

In model identification, lag length criteria indicate the maximum lag to test for and then various information criteria for all lags up to the specified maximum is shown. If the lag length is too short, autocorrelation of the error terms could lead to apparently significant and inefficient estimators. Therefore, one would receive erroneous results. Hannan-Quinn Information Criterion (HQ) tests can be habituated to determine the optimal lag number models and given as:

\[ HQ = -2L_{\text{max}} + 2k \log n \]  

(6)

Where \( L_{\text{max}} \) is the log-likelihood, \( k \) is the number of parameters and \( n \) is the number of observations.

The parameters of the VAR model need to be estimated once the number of lags in the model is determined. Although the structure of the VAR model looks very complex, the estimation of the parameters is not difficult as stated in [11]. The most common method is Ordinary Least Square Estimator (OLS), [14]. The OLS method is used to estimate the parameters since it is the natural estimator as stated by [15]. The basic condition with OLS approach demonstrates that how each independent variable influenced the dependent variable and can be written as:

\[ Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t \]  

(7)

Where \( \beta_0 \) indicates drift component, \( Y_t \) is dependent value, \( X_t \) is independent value and \( \varepsilon_t \) is white noise error.

The roots view exhibits the inverse roots of the AR characteristic polynomial. The roots might be shown as a diagram or as a table. The diagram view plots the roots in the complex plane where the horizontal axis is the real part and the vertical axis is the imaginary part of each root. Meanwhile, table view displays all roots in order of decreasing modulus, see [16]. The estimated VAR is stable on the off chance if all roots have modulus less than one and lie inside the unit circle. If the VAR is not stable, certain results such as impulse response standard errors are not valid.
According to [3], the VAR is normally used for forecasting systems of interrelated time series and for breaking down the dynamic impact of random disturbances on the system of variables. For single lag specification, the initial observation in the forecast sample will use the actual value of lagged \( Y \). Thus, if \( S \) is the first observation in the forecast sample, it will compute:

\[
y_s = \hat{c}_1 + \hat{c}_2 x_s + \hat{c}_3 z_s + \hat{c}_4 y_{s-1}
\]

(8)

Where \( y_{s-1} \) is the value of the lagged endogenous variable in the period prior to the start of the forecast sample and \( \hat{c} \) is the coefficient value for each model. This is the one-step ahead forecast.

2.3 Measurement Forecast Accuracy

Forecast evaluation is relevant to the forecaster when deciding on a model specification for subsequent use. The inclinations or misfortune capacity of the forecast user is key to the selection of the accuracy criterion by referring to [21]. Now consider the problem of measuring forecasting errors. This study denotes the actual value of the variable of interest in time period \( t \) as \( y_t \), and the predicted value as \( \hat{y}_t \). Then subtract the predicted value of \( \hat{y}_t \) from the actual value \( y_t \) to obtain forecast error.

The measurement of forecast accuracy, Mean Absolute Percentage Error (MAPE) will be employed in this study. It provides a measure of the distance of the true from the forecast value, see [22]. Suppose the forecast sample is \( j = T+1, T+2, \ldots, T+h \), and \( y_t \) denote the actual and forecast value in period \( t \) as \( \hat{y}_t \), respectively. The forecast evaluation measures are defined as:

\[
MAPE = 100 \times \frac{1}{h} \sum_{t=T+1}^{T+h} \left| \frac{y_t - \hat{y}_t}{y_t} \right|
\]

(9)

3.0 Results and Discussion

3.1 Correlation Test

Economists usually measure economic growth in terms of gross domestic product (GDP). GDP is calculated from a country's national accounts which report a yearly information on incomes, expenditure and investment for each sector of the economy. In this study, the significance correlation among the variables is examined over the period of the year from 1990 to 2015. The purpose of this correlation test is to verify whether all the variables that had been selected for this study are correlated to GDP or not and therefore can be used as the variables to forecast economic growth.

Table 1: Correlation Coefficient Analysis

<table>
<thead>
<tr>
<th>Economic indicator</th>
<th>GDP</th>
<th>CIC</th>
<th>EXC</th>
<th>EXT</th>
<th>RM</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIC</td>
<td>0.9868</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EXC</td>
<td>0.3806</td>
<td>0.3707</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0551)**</td>
<td>(0.0623)**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EXT</td>
<td>0.9717</td>
<td>0.9383</td>
<td>0.3194</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00)*</td>
<td>(0.00)*</td>
<td>(0.1117)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1 shows that CIC and EXT have a positive strong correlation with 0.9868 and 0.9717 at 1% significance level, respectively. In addition, RM also have a positive correlation with 0.8816 at 1% significance level. As for EXC, it displays a positive weak correlation of 0.3806 and is significant at level of 5%. It can be concluded that Currency in Circulation, External Reserve, Reserve Money and Exchange Rate are correlated towards Gross Domestic Product and can be used to forecast economic growth.

3.2 Unit Root test

The order of integration of each of the variable is inspected using the Augmented Dickey-Fuller (ADF) unit root test. Before conducting this test, an informal test for trends in the logarithmic levels and differences is done. The variables in the study were transformed into log form for carrying out the research. Data transformations are commonly used tools that can serve many functions in quantitative analysis of data. Applying the log transformation makes the data more normal as stated by [23]. Table 2 presents the results of the unit root tests for the four variables. The Null hypothesis is that series is non-stationary, or contain a unit root. The rejection of the null hypothesis based on the Mackinnon critical values. Note: ***, ** and * denotes significant at 1%, 5% and 10% significance level, respectively. It can be observed that all the variables (Currency In circulation, Exchange Rate, External Reserve and Reserve Money) exhibit non-stationary series which are integrated of first order. These result are consistent with the norm of macroeconomic series being I(1).

Table 2: Result of unit root test using ADF test

<table>
<thead>
<tr>
<th>Variable</th>
<th>Augmented Dickey Fuller (ADF)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
</tr>
<tr>
<td>Log CIC</td>
<td>Constant 3.7415</td>
</tr>
<tr>
<td></td>
<td>Constant With Trend 1.0935</td>
</tr>
<tr>
<td>Log EXC</td>
<td>-1.2673</td>
</tr>
<tr>
<td></td>
<td>0.8104</td>
</tr>
<tr>
<td>Log EXT</td>
<td>-1.3326</td>
</tr>
<tr>
<td></td>
<td>-1.7910</td>
</tr>
<tr>
<td>Log RM</td>
<td>1.0471</td>
</tr>
<tr>
<td></td>
<td>-0.5709</td>
</tr>
</tbody>
</table>

First Difference

| Log CIC   | -2.4268***                  |
|           | -5.2536***                  |
| Log EXC   | -6.1676***                  |
|           | -6.5848***                  |
| Log EXT   | -3.4606***                  |
|           | -4.0011***                  |
| Log RM    | -4.4321***                  |
|           | -3.8603***                  |

3.4 ARIMA Modeling

The model checking was done with ADF unit root test on CIC, EXC, EXT and RM. The outcome confirms that the series becomes stationary after the first-difference in the series (Table 2). Then lagging order number $p$ and $q$ is defined, where ARMA $(p, q)$ model’s order number can be judged using cutoff property of the model sample’s ACF and PACF. Each variable will be estimated using ARIMA approach and is discussed in the next subsection below.
3.4.1 Model Identification

According to [24], ARIMA \((p,d,q)\) parameters are rarely taken to exceed 2. So in this study, ARIMA models with up to 2 will be included and a total of 4 models will be estimated. Table 3 below shows different parameters of \(p\) and \(q\) among the several ARIMA model experimental upon. The lowest model based on AIC is selected and experimental results for model selection of each CIC, EXC, EXT and RM are shown as below.

**Table 3: Statistical Result of Different ARIMA Parameters for CIC, EXC, EXT and RM**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>ARIMA Model ((p,d,q))</th>
<th>AIC Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIC</td>
<td>(1,1,2)</td>
<td>17.5917</td>
</tr>
<tr>
<td>EXC</td>
<td>(1,1,1)</td>
<td>-1.9230</td>
</tr>
<tr>
<td>EXT</td>
<td>(1,1,0)</td>
<td>20.9876</td>
</tr>
<tr>
<td>RM</td>
<td>(2,1,2)</td>
<td>19.1124</td>
</tr>
</tbody>
</table>

3.4.2 Parameter Estimation

From the Table 3, the fitted ARIMA model are as follows:

\[
CIC = 257.4820 - 0.1220Y_{t-1} + \varepsilon_t - 0.2328\varepsilon_{t-1} - 0.2226\varepsilon_{t-2}
\]

\[
EXC = -0.0020 - 0.7462Y_{t-1} + \varepsilon_t + 0.0082\varepsilon_{t-1}
\]

\[
EXT = 1633.12 + 0.2483Y_{t-1} + \varepsilon_t
\]

\[
RM = 256.20 + 0.0045Y_{t-1} - 0.9957Y_{t-2} + \varepsilon_t - 0.0371\varepsilon_{t-1} + 0.9999\varepsilon_{t-2}
\]

3.4.3 Model Diagnostic

Result indicates that all the values are less than one. In other words, the eigen values of the system obtainable in modulus lies within the unit circle. Hence, it can be concluded that all ARIMA for each model satisfies the stability condition.

3.4.4 Forecasting

This study then focus on the purpose of this study to forecast economic growth using CIC, EXC, EXT and RM. Figure 1 gives graphical illustration of the predicted values against actual values to see the performance of the ARIMA model selected. The current data is starting from January 1998 until January 2016. Hence, the predicted years start from February 2016 until January 2020.
3.5 VAR Modeling

As shown in Table 2, all the time series are non-stationary. Here, again, the variables checking were done with the Augmented Dickey-Fuller (ADF) unit root test. The results confirm that the series becomes stationary after the first-difference of the series as shown in Table 2.

3.5.1 Parameter Estimation

As all variables are integrated of order 1, then we can apply Johansen approach to determine maximum possible cointegrating relationships when we have more than two variables. Both result from Trace test and Maximum Eigenvalue test showed that there is no cointegration exist between the CIC, EXC, EXT and RM since the probability value is greater than 0.05. We conclude that there is no exist some long run equilibrium relationship between the variables. Therefore, we can estimate the VAR model since the variables are not cointegrated.

Based on HQ criteria, see [25], the chosen number of lags is 2. Then there are 36 parameters to be estimated. Therefore, the estimated VAR model can be expressed as following:

\[
\text{Currency in Circulation} = 0.4072 \text{CIC}_{t-1} + 0.5850 \text{CIC}_{t-2} - 411.5426 \text{EXC}_{t-1} + 1188.3076 \text{EXC}_{t-2} - 0.0187 \text{EXT}_{t-1} + 0.0219 \text{EXT}_{t-2} + 0.2139 \text{RM}_{t-1} - 0.2124 \text{RM}_{t-2} - 3049.3478
\]
Exchange Rate = 3.8432*CIC_{t-1} - 5.3831*CIC_{t-2} + 1.0281*EXC_{t-1} - 0.0840*EXC_{t-2} - 1.2573*EXT_{t-1} + 1.0765*EXT_{t-2} - 2.1541*RM_{t-1} + 1.73434*RM_{t-2} + 0.20911

External Reserve = 0.00890*CIC_{t-1} + 0.1917*CIC_{t-2} + 5570.9826*EXC_{t-1} - 7822.5786*EXC_{t-2} + 1.2371*EXT_{t-1} - 0.2561*EXT_{t-2} - 0.1591*RM_{t-1} + 0.0801*RM_{t-2} + 12085.9240

Reserve Money = -0.8554*IC_{t-1} + 0.9010*IC_{t-2} - 2636.0999*EXC_{t-1} + 2557.2739*EXC_{t-2} - 0.00830*EXT_{t-1} + 0.0131*EXT_{t-2} + 1.3044*RM_{t-1} - 0.3341*RM_{t-2} - 255.5732

3.5.2 Model Diagnostic

The result indicates that all the Eigen values of the system obtainable in modulus lies within the unit circle. Hence, it can be concluded that the VAR model satisfies the stability condition (Figure 2).

![Inverse Roots of AR Characteristic Polynomial](image)

Figure 2: Results of Characteristics Polynomial Roots

3.5.3 Forecasting

As mentioned before, the VAR model is valid and the residuals follow white noise assumption. Then, we apply the VAR model above to predict the economic growth in Malaysia during February 2016 to December 2020 by using the multi-step ahead forecast method. The forecasting results are presented in Figure 3. From the figure, the predicted values of CIC, EXT and RM show an upward trend. Meanwhile, in EXC, the graph shows a tendency of increase for the period of 17 years from 2004 to 2020.
3.6 Comparative Study

This section will discuss about the comparative study of the model performance. The proposed models are VAR (2) for multivariate forecasting and ARIMA for univariate forecasting. ARIMA model that has been fitted are (1,1,2) for CIC, (1,1,1) for EXC, (1,1,0) for EXT and (2,1,2) for RM.

After calculation as mentioned in (9), the evaluation of the forecast results developed by time series ARIMA and VAR are presented in Table 4. The best model that can be used in forecasting CIC, EXC, EXT and RM price with a certain degree of accuracy is chosen.

Table 4: Accuracy Measurement for each model

<table>
<thead>
<tr>
<th>MODEL</th>
<th>ARIMA</th>
<th>VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIC</td>
<td>40.4422</td>
<td>7.4881</td>
</tr>
<tr>
<td>EXC</td>
<td>6.5919</td>
<td>7.8627</td>
</tr>
<tr>
<td>EXT</td>
<td>17.3926</td>
<td>14.7080</td>
</tr>
<tr>
<td>RM</td>
<td>28.6907</td>
<td>11.1232</td>
</tr>
</tbody>
</table>
From the Table 4, it is clearly seen that VAR give the best result for predicting CIC, EXT and RM in terms of less error. Meanwhile, the ARIMA model seems to be the best for predicting EXC.

4.0 Conclusion

The results from the models VAR provide efficient outcomes except for forecasting EXC. The value of MAPE for the VAR model is significantly smaller than the ARIMA model for forecasting CIC, EXT and RM. In conclusion, this study reveals that the multivariate model outperform univariate model in term of statistical results and forecasting values for predicting CIC, EXT and RM.

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